

Students write an equation of the form $y = ab^x$ that goes through two given points. (Equations of this form have an asymptote at $y = 0$.)

Example 1

Find a possible equation for an exponential function that passes through the points $(0, 8)$ and $(4, \frac{1}{2})$.

Solution: Substitute the x - and y -coordinates of each pair of points into the general equation. Then solve the resulting system of two equations to determine a and b .

$$y = ab^x$$

$$\text{Since } (x, y) = (0, 8)$$

$$8 = ab^0$$

$$\text{Since } b^0 = 1,$$

$$8 = a(1), \text{ or}$$

$$a = 8.$$

$$y = ab^x$$

$$\text{Since } (x, y) = (4, \frac{1}{2}),$$

$$\frac{1}{2} = ab^4$$

Substituting $a = 8$ from the first equation into $\frac{1}{2} = ab^4$ from the second equation,

$$\frac{1}{2} = ab^4$$

$$\text{But, } a = 8,$$

$$\frac{1}{2} = 8b^4$$

$$\frac{1}{16} = b^4$$

$$\sqrt[4]{\frac{1}{16}} = \sqrt[4]{b^4}$$

$$\frac{1}{2} = b$$

Since a and b have been determined, we can now write the equation:

$$y = 8\left(\frac{1}{2}\right)^x$$

Example 2

In the year 2000, Club Leopard was first introduced on the Internet. In 2004, it had 14,867 “leopards” (members). In 2007, the leopard population had risen to 22,610. Model this data with an exponential function and use the model to predict the leopard population in the year 2012.

Solution: We can call the year 2000 our time zero, or $x = 0$. Then 2004 is $x = 4$, and the year 2007 will be $x = 7$. This gives us two data points, (4, 14867) and (7, 22610).

To model with an exponential function we will use the equation $y = ab^x$ and substitute both coordinate pairs to obtain a system of two equations.

$$\begin{array}{ll} y = ab^x & y = ab^x \\ 14867 = ab^4 & 22610 = ab^7 \end{array}$$

Preparing to use the Equal Values Method to solve the system of equations, we rewrite both equations in “a=” form:

$$\begin{array}{ll} 14867 = ab^4 & 22610 = ab^7 \\ a = \frac{14867}{b^4} & a = \frac{22610}{b^7} \end{array}$$

Then by the Equal Values Method,

$$\begin{aligned} \frac{14867}{b^4} &= \frac{22610}{b^7} \\ 14867b^7 &= 22610b^4 \\ \frac{b^7}{b^4} &= \frac{22610}{14867} \\ b^3 &= \frac{22610}{14867} \\ \sqrt[3]{b^3} &= \sqrt[3]{\frac{22610}{14867}} \\ b &\approx 1.15 \end{aligned}$$

From the equations above,

$$\begin{aligned} a &= \frac{14867}{b^4} \\ \text{Since } b &\approx 1.15, \\ a &= \frac{14867}{(1.15)^4} \\ a &\approx 8500 \end{aligned}$$

Since $a \approx 8500$ and $b \approx 1.15$ we can write the equation: $y = 8500 \cdot 1.15^x$, where y represents the number of members, and x represents the number of years since 2000.

We will use the equation with $x = 12$ to predict the population in 2012.

$$\begin{aligned} y &= 8500(1.15)^x \\ y &= 8500(1.15)^{12} \\ y &\approx 45477 \end{aligned}$$

Assuming the trend continues to the year 2012 as it has in the past, we predict the population in 2012 to be 45,477.

Problems

For each of the following pairs of points, find the equation of an exponential function with an asymptote $y = 0$ that passes through them.

- (0, 6) and (3, 48)
- (1, 21) and (2, 147)
- (-1, 72.73) and (3, 106.48)
- (-2, 351.5625) and (3, 115.2)
- On a cold wintry day the temperature outside hovered at 0°F . Karen made herself a cup of cocoa, and took it outside where she would be chopping some wood. However, she decided to conduct a mini science experiment instead of drinking her cocoa, so she placed a thermometer in the cocoa and left it sitting next to her as she worked. She wrote down the time and the reading on the thermometer as shown in the table below.

Time since 1st reading	5	10	12	15
Temp ($^{\circ}\text{C}$)	24.41°	8.51°	5.58°	2.97°

Find the equation of an exponential function of the form $y = ab^x$ that models this data.

Answers

- $y = 6(2)^x$
- $y = 3(7)^x$
- $y = 80(1.1)^x$
- $y = 225(0.8)^x$
- Answers will vary, but should be close to $y = 70(0.81)^x$.